Eletromagnetismo

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Lista - 2

2.1 - Mostrar que a seguinte medida é invariante

$$\int \frac{d^3\mathbf{p}}{2p_0} \quad \text{onde:} \quad p_0^2 = \mathbf{p}^2 + m^2 \tag{1}$$

onde $\mathbf{v}$ e $\mathbf{v}'$ - velocidade da partícula no referencial K e K', $\mathbf{V}$ - velocidade do referencial K em relação a K'.

....:: Solução ::..

$$\int \frac{d^3\mathbf{p}}{2p_0} = \int d^4p\delta(p^2 - m^2)\Theta(p_0) \quad \textit{onde} \quad p^2 = p_\mu p^\mu = p_0^2 - \mathbf{p}^2 \quad \textit{e} \quad p_0 = E = \pm\sqrt{\mathbf{p}^2 + m^2} \tag{2}$$

$$\textit{função de Heaviside:} \quad \Theta(\alpha) = \begin{cases} 1, & \text{se} \quad \alpha > 0 \\ 0, & \text{se} \quad \alpha < 0 \end{cases} \tag{3}$$

$$\begin{aligned} \int d^4p\delta(p^2 - m^2)\Theta(p_0) &= \int d^4p\delta(p_0^2 - \mathbf{p}^2 - m^2)\Theta(p_0) = \int d^4p\delta\Big[p_0^2 - (\mathbf{p}^2 + m^2)\Big]\Theta(p_0) \\ &= \int d^4p\delta(p_0^2 - E^2)\Theta(p_0) \end{aligned} \tag{4}$$

$$\textit{propriedade da função delta:} \quad \delta(x^2 - a^2) = \frac{1}{|2a|}\Big[\delta(x + a) + \delta(x - a)\Big] \tag{5}$$

das eqs.(3), (4) e (5)

$$\begin{aligned} \int d^4p\delta(p_0^2 - E^2)\Theta(p_0) &= \int \frac{d^4p}{2E}\Big[\delta(p_0 + E) + \delta(p_0 - E)\Big]\Theta(p_0) \\ &= \int \frac{d^4p}{2E}\underbrace{\delta(p_0 + E)\Theta(p_0)}_{\Theta(p_0)=0} + \int \frac{d^4p}{2E}\delta(p_0 - E)\Theta(p_0) \\ &\qquad \textit{pois} \quad p_0 = -E \ (E > 0 \textit{ é positivo definido, partícula livre}) \\ &= \int \frac{d^3\mathbf{p}}{2E}dp_0\underbrace{\delta(p_0 - E)\Theta(p_0)}_{\Theta(p_0)=1} = \int \frac{d^3\mathbf{p}}{2p_0} \\ &\qquad \text{pois} \quad p_0 = E \end{aligned} \tag{6}$$

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2.2 - Demonstrar a relação

$$v = \frac{\sqrt{(\mathbf{v}' + \mathbf{V})^2 - \frac{1}{c^2}(\mathbf{v}' \times \mathbf{V})^2}}{1 + \frac{(\mathbf{v}' \cdot \mathbf{V})}{c^2}} \tag{7}$$

....:: Solução ::...
Suponha um sistema K que se move em relação ao sistema K' com velocidade V ao longo do eixo x.

Transformções de Lorentz,

$$\begin{aligned} x &= \gamma(x' + Vt') \quad \longrightarrow \quad dx = \gamma(dx' + Vdt'), && (8)\\ y &= y' \quad \longrightarrow \quad dy = dy', && (9)\\ z &= z' \quad \longrightarrow \quad dz = dz', && (10)\\ t &= \gamma(t' + V/c^2 x') \quad \longrightarrow \quad dt = \gamma(dt' + V/c^2 dx'). && (11) \end{aligned}$$

onde $\mathbf{V} = (V_x, 0, 0)$ é velocidade do sistema K em relação ao sistema K'

Dividindo as eqs, (8),(9) e (10) pela eq. (11) temos,

$$\begin{aligned} \mathbf{v} &= \frac{d\mathbf{r}}{dt}, \text{ \textit{velocidade da partícula no sistema} } K && (12)\\ \mathbf{v}' &= \frac{d\mathbf{r}'}{dt'}, \text{ \textit{velocidade da partícula no sistema} } K' && (13)\\ & && (14) \end{aligned}$$

cuja as componentes são,

$$v_x = \frac{v'_x + V_x}{1 + \frac{v'_x V_x}{c^2}} \quad v_y = \frac{v'_y\sqrt{1 - \frac{V_x^2}{c^2}}}{1 + \frac{v'_x V_x}{c^2}} \quad v_z = \frac{v'_z\sqrt{1 - \frac{V_x^2}{c^2}}}{1 + \frac{v'_x V_x}{c^2}} \tag{15}$$

$$\text{fazendo: } \gamma = \frac{1}{\sqrt{1 - \frac{V_x^2}{c^2}}} \quad \text{e} \quad a \equiv \frac{1}{1 + \frac{v'_x V_x}{c^2}} = \frac{1}{1 + \frac{\mathbf{v}' \cdot \mathbf{V}}{c^2}} \tag{16}$$

calculando o módulo de $\mathbf{v}$,

$$
\begin{aligned}
v &= |\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \\
&= \sqrt{a^2(v'_x + V_x)^2 + a^2(\gamma^{-1}v'_y)^2 + a^2(\gamma^{-1}v'_z)^2} \\
&= a\sqrt{v'^2_x + V_x^2 + 2v'_xV_x + \gamma^{-2}(v'^2_y + v'^2_z)} \\
&= a\sqrt{v'^2_x + V_x^2 + 2v'_xV_x + (1 - \frac{V_x^2}{c^2})(v'^2_y + v'^2_z)} \\
&= a\sqrt{v'^2_x + v'^2_y + v'^2_z + V_x^2 + 2v'_xV_x - \frac{V_x^2}{c^2}(v'^2_y + v'^2_z)} \\
&= a\sqrt{\mathbf{v}'^2 + \mathbf{V}^2 + 2\mathbf{v}' \cdot \mathbf{V} - \frac{1}{c^2}(\mathbf{v}' \times \mathbf{V})^2} \\
&= \frac{\sqrt{(\mathbf{v}' + \mathbf{V})^2 - \frac{1}{c^2}(\mathbf{v}' \times \mathbf{V})^2}}{1 + \frac{\mathbf{v}' \cdot \mathbf{V}}{c^2}} \qquad (17)
\end{aligned}
$$

2.3 - Sejam os tensores simétricos $S_{\mu\nu}$ e antisimétrico $F_{\mu\nu}$. Obtenha as regras de transformação para as componentes destes tensores com as transformações de Lorentz no plano (t, x).

....:: Solução ::..

$$
\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad e \quad \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} \qquad (18)
$$

Para o tensor simétrico $S'_{\mu\nu}$,

$$
\begin{aligned}
S'_{\mu\nu} &= \Lambda^\alpha_\mu \Lambda^\beta_\nu S_{\alpha\beta} \qquad (19) \\
&= \Lambda^0_\mu\Lambda^0_\nu S_{00} + \Lambda^0_\mu\Lambda^1_\nu S_{01} + \Lambda^0_\mu\Lambda^2_\nu S_{02} + \Lambda^0_\mu\Lambda^3_\nu S_{03} \\
&+ \Lambda^1_\mu\Lambda^0_\nu S_{10} + \Lambda^1_\mu\Lambda^1_\nu S_{11} + \Lambda^1_\mu\Lambda^2_\nu S_{12} + \Lambda^1_\mu\Lambda^3_\nu S_{13} \\
&+ \Lambda^2_\mu\Lambda^0_\nu S_{20} + \Lambda^2_\mu\Lambda^1_\nu S_{21} + \Lambda^2_\mu\Lambda^2_\nu S_{22} + \Lambda^2_\mu\Lambda^3_\nu S_{23} \\
&+ \Lambda^3_\mu\Lambda^0_\nu S_{30} + \Lambda^3_\mu\Lambda^1_\nu S_{31} + \Lambda^3_\mu\Lambda^2_\nu S_{32} + \Lambda^3_\mu\Lambda^3_\nu S_{33}
\end{aligned}
$$

fixando $\mu = 0$ *e fazendo variar* $\nu = 0, \dots, 3$
para $\nu = 0$

$$
\begin{aligned}
S'_{00} &= \Lambda^0_0\Lambda^0_0 S_{00} + \Lambda^0_0\Lambda^1_0 S_{01} + \Lambda^0_0\Lambda^2_0 S_{02} + \Lambda^0_0\Lambda^3_0 S_{03} \\
&+ \Lambda^1_0\Lambda^0_0 S_{10} + \Lambda^1_0\Lambda^1_0 S_{11} + \Lambda^1_0\Lambda^2_0 S_{12} + \Lambda^1_0\Lambda^3_0 S_{13} \\
&+ \Lambda^2_0\Lambda^0_0 S_{20} + \Lambda^2_0\Lambda^1_0 S_{21} + \Lambda^2_0\Lambda^2_0 S_{22} + \Lambda^2_0\Lambda^3_0 S_{23} \\
&+ \Lambda^3_0\Lambda^0_0 S_{30} + \Lambda^3_0\Lambda^1_0 S_{31} + \Lambda^3_0\Lambda^2_0 S_{32} + \Lambda^3_0\Lambda^3_0 S_{33} \\
&= \Lambda^0_0\Lambda^0_0 S_{00} + \Lambda^0_0\Lambda^1_0 S_{01} + \Lambda^1_0\Lambda^0_0 S_{10} + \Lambda^1_0\Lambda^1_0 S_{11} \\
S'_{00} &= \gamma^2\beta\left[\frac{1}{\beta}S_{00} + 2\,S_{01} + \beta S_{11}\right] \qquad (20)
\end{aligned}
$$

para $\nu = 1$

$$\begin{aligned}
S'_{01} &= \Lambda^0_0\Lambda^0_1 S_{00} + \Lambda^0_0\Lambda^1_1 S_{01} + \Lambda^0_0\Lambda^2_1 S_{02} + \Lambda^0_0\Lambda^3_1 S_{03} \\
&+ \Lambda^1_0\Lambda^0_1 S_{10} + \Lambda^1_0\Lambda^1_1 S_{11} + \Lambda^1_0\Lambda^2_1 S_{12} + \Lambda^1_0\Lambda^3_1 S_{13} \\
&+ \Lambda^2_0\Lambda^0_1 S_{20} + \Lambda^2_0\Lambda^1_1 S_{21} + \Lambda^2_0\Lambda^2_1 S_{22} + \Lambda^2_0\Lambda^3_1 S_{23} \\
&+ \Lambda^3_0\Lambda^0_1 S_{30} + \Lambda^3_0\Lambda^1_1 S_{31} + \Lambda^3_0\Lambda^2_1 S_{32} + \Lambda^3_0\Lambda^3_1 S_{33} \\
&= \Lambda^0_0\Lambda^0_1 S_{00} + \Lambda^0_0\Lambda^1_1 S_{01} + \Lambda^1_0\Lambda^0_1 S_{10} + \Lambda^1_0\Lambda^1_1 S_{11} \\
S'_{01} &= \gamma^2\beta\Big[S_{00} + (1+\beta)S_{01} + S_{11}\Big] \qquad (21)
\end{aligned}$$

para $\nu = 2$

$$\begin{aligned}
S'_{02} &= \Lambda^0_0\Lambda^0_2 S_{00} + \Lambda^0_0\Lambda^1_2 S_{01} + \Lambda^0_0\Lambda^2_2 S_{02} + \Lambda^0_0\Lambda^3_2 S_{03} \\
&+ \Lambda^1_0\Lambda^0_2 S_{10} + \Lambda^1_0\Lambda^1_2 S_{11} + \Lambda^1_0\Lambda^2_2 S_{12} + \Lambda^1_0\Lambda^3_2 S_{13} \\
&+ \Lambda^2_0\Lambda^0_2 S_{20} + \Lambda^2_0\Lambda^1_2 S_{21} + \Lambda^2_0\Lambda^2_2 S_{22} + \Lambda^2_0\Lambda^3_2 S_{23} \\
&+ \Lambda^3_0\Lambda^0_2 S_{30} + \Lambda^3_0\Lambda^1_2 S_{31} + \Lambda^3_0\Lambda^2_2 S_{32} + \Lambda^3_0\Lambda^3_2 S_{33} \\
&= \Lambda^0_0\Lambda^2_2 S_{02} + \Lambda^1_0\Lambda^2_2 S_{12} \\
S'_{02} &= \gamma\Big[S_{02} + \beta S_{12}\Big] \qquad (22)
\end{aligned}$$

para $\nu = 3$

$$\begin{aligned}
S'_{03} &= \Lambda^0_0\Lambda^0_3 S_{00} + \Lambda^0_0\Lambda^1_3 S_{01} + \Lambda^0_0\Lambda^2_3 S_{02} + \Lambda^0_0\Lambda^3_3 S_{03} \\
&+ \Lambda^1_0\Lambda^0_3 S_{10} + \Lambda^1_0\Lambda^1_3 S_{11} + \Lambda^1_0\Lambda^2_3 S_{12} + \Lambda^1_0\Lambda^3_3 S_{13} \\
&+ \Lambda^2_0\Lambda^0_3 S_{20} + \Lambda^2_0\Lambda^1_3 S_{21} + \Lambda^2_0\Lambda^2_3 S_{22} + \Lambda^2_0\Lambda^3_3 S_{23} \\
&+ \Lambda^3_0\Lambda^0_3 S_{30} + \Lambda^3_0\Lambda^1_3 S_{31} + \Lambda^3_0\Lambda^2_3 S_{32} + \Lambda^3_0\Lambda^3_3 S_{33} \\
&= \Lambda^0_0\Lambda^3_3 S_{03} + \Lambda^1_0\Lambda^3_3 S_{13} \\
S'_{03} &= \gamma\Big[S_{03} + \beta S_{13}\Big] \qquad (23)
\end{aligned}$$

fixando $\mu = 1$ *e fazendo variar* $\nu = 0, \ldots, 3$
para $\nu = 0$

$$S'_{10} = S'_{01} = \gamma^2\beta\Big[S_{00} + (1+\beta)S_{01} + S_{11}\Big]$$

para $\nu = 1$

$$\begin{aligned}
S'_{11} &= \Lambda^0_1\Lambda^0_1 S_{00} + \Lambda^0_1\Lambda^1_1 S_{01} + \Lambda^0_1\Lambda^2_1 S_{02} + \Lambda^0_1\Lambda^3_1 S_{03} \\
&+ \Lambda^1_1\Lambda^0_1 S_{10} + \Lambda^1_1\Lambda^1_1 S_{11} + \Lambda^1_1\Lambda^2_1 S_{12} + \Lambda^1_1\Lambda^3_1 S_{13} \\
&+ \Lambda^2_1\Lambda^0_1 S_{20} + \Lambda^2_1\Lambda^1_1 S_{21} + \Lambda^2_1\Lambda^2_1 S_{22} + \Lambda^2_1\Lambda^3_1 S_{23} \\
&+ \Lambda^3_1\Lambda^0_1 S_{30} + \Lambda^3_1\Lambda^1_1 S_{31} + \Lambda^3_1\Lambda^2_1 S_{32} + \Lambda^3_1\Lambda^3_1 S_{33} \\
&= \Lambda^0_1\Lambda^0_1 S_{00} + \Lambda^0_1\Lambda^1_1 S_{01} + \Lambda^1_1\Lambda^0_1 S_{10} + \Lambda^1_1\Lambda^1_1 S_{11} \\
S'_{11} &= \gamma^2\beta^2\Big[S_{00} - 2\beta S_{01} + \frac{1}{\beta^2}S_{11}\Big] \qquad (24)
\end{aligned}$$

para $\nu = 2$

$$\begin{aligned}
S'_{12} &= \Lambda^0_1\Lambda^0_2 S_{00} + \Lambda^0_1\Lambda^1_2 S_{01} + \Lambda^0_1\Lambda^2_2 S_{02} + \Lambda^0_1\Lambda^3_2 S_{03} \\
&+ \Lambda^1_1\Lambda^0_2 S_{10} + \Lambda^1_1\Lambda^1_2 S_{11} + \Lambda^1_1\Lambda^2_2 S_{12} + \Lambda^1_1\Lambda^3_2 S_{13} \\
&+ \Lambda^2_1\Lambda^0_2 S_{20} + \Lambda^2_1\Lambda^1_2 S_{21} + \Lambda^2_1\Lambda^2_2 S_{22} + \Lambda^2_1\Lambda^3_2 S_{23} \\
&+ \Lambda^3_1\Lambda^0_2 S_{30} + \Lambda^3_1\Lambda^1_2 S_{31} + \Lambda^3_1\Lambda^2_2 S_{32} + \Lambda^3_1\Lambda^3_2 S_{33} \\
&= \Lambda^0_1\Lambda^2_2 S_{02} + \Lambda^1_1\Lambda^2_2 S_{12} \\
S'_{12} &= \gamma^2 S_{12} - \gamma S_{02}
\end{aligned}$$

para $\nu = 3$

$$\begin{aligned}
S'_{13} &= \Lambda_1^0\Lambda_3^0 S_{00} + \Lambda_1^0\Lambda_3^1 S_{01} + \Lambda_1^0\Lambda_3^2 S_{02} + \Lambda_1^0\Lambda_3^3 S_{03} \\
&+ \Lambda_1^1\Lambda_3^0 S_{10} + \Lambda_1^1\Lambda_3^1 S_{11} + \Lambda_1^1\Lambda_3^2 S_{12} + \Lambda_1^1\Lambda_3^3 S_{13} \\
&+ \Lambda_1^2\Lambda_3^0 S_{20} + \Lambda_1^2\Lambda_3^1 S_{21} + \Lambda_1^2\Lambda_3^2 S_{22} + \Lambda_1^2\Lambda_3^3 S_{23} \\
&+ \Lambda_1^3\Lambda_3^0 S_{30} + \Lambda_1^3\Lambda_3^1 S_{31} + \Lambda_1^3\Lambda_3^2 S_{32} + \Lambda_1^3\Lambda_3^3 S_{33} \\
&= \Lambda_1^0\Lambda_3^3 S_{03} + \Lambda_1^1\Lambda_3^3 S_{13} \\
S'_{13} &= \gamma S_{13} - \gamma^2\beta S_{03}
\end{aligned}$$

fixando $\mu = 2$ *e fazendo variar* $\nu = 0, ..., 3$
para $\nu = 0$

$$S'_{20} = S'_{02} = \gamma\Big[S_{02} + \beta S_{12}\Big] \tag{25}$$

para $\nu = 1$

$$S'_{21} = S'_{12} = \gamma^2 S_{12} - \gamma S_{02}$$

para $\nu = 2$

$$\begin{aligned}
S'_{22} &= \Lambda_2^0\Lambda_2^0 S_{00} + \Lambda_2^0\Lambda_2^1 S_{01} + \Lambda_2^0\Lambda_2^2 S_{02} + \Lambda_2^0\Lambda_2^3 S_{03} \\
&+ \Lambda_2^1\Lambda_2^0 S_{10} + \Lambda_2^1\Lambda_2^1 S_{11} + \Lambda_2^1\Lambda_2^2 S_{12} + \Lambda_2^1\Lambda_2^3 S_{13} \\
&+ \Lambda_2^2\Lambda_2^0 S_{20} + \Lambda_2^2\Lambda_2^1 S_{21} + \Lambda_2^2\Lambda_2^2 S_{22} + \Lambda_2^2\Lambda_2^3 S_{23} \\
&+ \Lambda_2^3\Lambda_2^0 S_{30} + \Lambda_2^3\Lambda_2^1 S_{31} + \Lambda_2^3\Lambda_2^2 S_{32} + \Lambda_2^3\Lambda_2^3 S_{33} \\
&= \Lambda_2^2\Lambda_2^2 S_{22} \\
S'_{22} &= S_{22}
\end{aligned}$$

para $\nu = 3$

$$\begin{aligned}
S'_{23} &= \Lambda_2^0\Lambda_3^0 S_{00} + \Lambda_2^0\Lambda_3^1 S_{01} + \Lambda_2^0\Lambda_3^2 S_{02} + \Lambda_2^0\Lambda_3^3 S_{03} \\
&+ \Lambda_2^1\Lambda_3^0 S_{10} + \Lambda_2^1\Lambda_3^1 S_{11} + \Lambda_2^1\Lambda_3^2 S_{12} + \Lambda_2^1\Lambda_3^3 S_{13} \\
&+ \Lambda_2^2\Lambda_3^0 S_{20} + \Lambda_2^2\Lambda_3^1 S_{21} + \Lambda_2^2\Lambda_3^2 S_{22} + \Lambda_2^2\Lambda_3^3 S_{23} \\
&+ \Lambda_2^3\Lambda_3^0 S_{30} + \Lambda_2^3\Lambda_3^1 S_{31} + \Lambda_2^3\Lambda_3^2 S_{32} + \Lambda_2^3\Lambda_3^3 S_{33} \\
&= \Lambda_2^2\Lambda_3^3 S_{23} \\
S'_{23} &= S_{23}
\end{aligned}$$

fixando $\mu = 3$ *e fazendo variar* $\nu = 0, ..., 3$
para $\nu = 1$

$$S'_{30} = S'_{03} = \gamma\Big[S_{03} + \beta S_{13}\Big] \tag{26}$$

para $\nu = 1$

$$S'_{31} = S'_{13} = \gamma S_{13} - \gamma^2\beta S_{03}$$

para $\nu = 2$

$$S'_{32} = S'_{23} = S_{23}$$

para $\nu = 3$

$$\begin{aligned}
S'_{33} &= \Lambda_3^0\Lambda_3^0 S_{00} + \Lambda_3^0\Lambda_3^1 S_{01} + \Lambda_3^0\Lambda_3^2 S_{02} + \Lambda_3^0\Lambda_3^3 S_{03} \\
&+ \Lambda_3^1\Lambda_3^0 S_{10} + \Lambda_3^1\Lambda_3^1 S_{11} + \Lambda_3^1\Lambda_3^2 S_{12} + \Lambda_3^1\Lambda_3^3 S_{13} \\
&+ \Lambda_3^2\Lambda_3^0 S_{20} + \Lambda_3^2\Lambda_3^1 S_{21} + \Lambda_3^2\Lambda_3^2 S_{22} + \Lambda_3^2\Lambda_3^3 S_{23} \\
&+ \Lambda_3^3\Lambda_3^0 S_{30} + \Lambda_3^3\Lambda_3^1 S_{31} + \Lambda_3^3\Lambda_3^2 S_{32} + \Lambda_3^3\Lambda_3^3 S_{33} \\
&= \Lambda_3^3\Lambda_3^3 S_{33} \\
S'_{33} &= S_{33}
\end{aligned}$$

Para o tensor antisimétrico $F'_{\mu\nu}$,

$$F'_{\mu\nu} = \Lambda_\mu^\alpha \Lambda_\nu^\beta F_{\alpha\beta} \tag{27}$$

fixando $\mu = 0$ *e fazendo variar* $\nu = 0, \ldots, 3$
para $\nu = 0$

$$\begin{aligned}
F'_{00} &= \Lambda_0^0\Lambda_0^0 F_{00} + \Lambda_0^0\Lambda_0^1 F_{01} + \Lambda_0^0\Lambda_0^2 F_{02} + \Lambda_0^0\Lambda_0^3 F_{03} \\
&+ \Lambda_0^1\Lambda_0^0 F_{10} + \Lambda_0^1\Lambda_0^1 F_{11} + \Lambda_0^1\Lambda_0^2 F_{12} + \Lambda_0^1\Lambda_0^3 F_{13} \\
&+ \Lambda_0^2\Lambda_0^0 F_{20} + \Lambda_0^2\Lambda_0^1 F_{21} + \Lambda_0^2\Lambda_0^2 F_{22} + \Lambda_0^2\Lambda_0^3 F_{23} \\
&+ \Lambda_0^3\Lambda_0^0 F_{30} + \Lambda_0^3\Lambda_0^1 F_{31} + \Lambda_0^3\Lambda_0^2 F_{32} + \Lambda_0^3\Lambda_0^3 F_{33} \\
&= \Lambda_0^0\Lambda_0^0 F_{00} + \Lambda_0^0\Lambda_0^1 F_{01} + \Lambda_0^1\Lambda_0^0 F_{10} + \Lambda_0^1\Lambda_0^1 F_{11} \\
F'_{00} &= 0 \qquad (28)
\end{aligned}$$

para $\nu = 1$

$$\begin{aligned}
F'_{01} &= \Lambda_0^0\Lambda_1^0 F_{00} + \Lambda_0^0\Lambda_1^1 F_{01} + \Lambda_0^0\Lambda_1^2 F_{02} + \Lambda_0^0\Lambda_1^3 F_{03} \\
&+ \Lambda_0^1\Lambda_1^0 F_{10} + \Lambda_0^1\Lambda_1^1 F_{11} + \Lambda_0^1\Lambda_1^2 F_{12} + \Lambda_0^1\Lambda_1^3 F_{13} \\
&+ \Lambda_0^2\Lambda_1^0 F_{20} + \Lambda_0^2\Lambda_1^1 F_{21} + \Lambda_0^2\Lambda_1^2 F_{22} + \Lambda_0^2\Lambda_1^3 F_{23} \\
&+ \Lambda_0^3\Lambda_1^0 F_{30} + \Lambda_0^3\Lambda_1^1 F_{31} + \Lambda_0^3\Lambda_1^2 F_{32} + \Lambda_0^3\Lambda_1^3 F_{33} \\
&= \Lambda_0^0\Lambda_1^1 F_{01} + \Lambda_0^1\Lambda_1^0 F_{10} \\
F'_{01} &= \gamma^2\Big[1 - \beta^2\Big] F_{01} \qquad (29)
\end{aligned}$$

para $\nu = 2$

$$\begin{aligned}
F'_{02} &= \Lambda_0^0\Lambda_2^0 F_{00} + \Lambda_0^0\Lambda_2^1 F_{01} + \Lambda_0^0\Lambda_2^2 F_{02} + \Lambda_0^0\Lambda_2^3 F_{03} \\
&+ \Lambda_0^1\Lambda_2^0 F_{10} + \Lambda_0^1\Lambda_2^1 F_{11} + \Lambda_0^1\Lambda_2^2 F_{12} + \Lambda_0^1\Lambda_2^3 F_{13} \\
&+ \Lambda_0^2\Lambda_2^0 F_{20} + \Lambda_0^2\Lambda_2^1 F_{21} + \Lambda_0^2\Lambda_2^2 F_{22} + \Lambda_0^2\Lambda_2^3 F_{23} \\
&+ \Lambda_0^3\Lambda_2^0 F_{30} + \Lambda_0^3\Lambda_2^1 F_{31} + \Lambda_0^3\Lambda_2^2 F_{32} + \Lambda_0^3\Lambda_2^3 F_{33} \\
&= \Lambda_0^0\Lambda_2^2 F_{02} + \Lambda_0^1\Lambda_2^2 F_{12} \\
F'_{02} &= \gamma\Big[F_{02} - \beta F_{12}\Big] \qquad (30)
\end{aligned}$$

para $\nu = 3$

$$\begin{aligned}
F'_{03} &= \Lambda_0^0\Lambda_3^0 F_{00} + \Lambda_0^0\Lambda_3^1 F_{01} + \Lambda_0^0\Lambda_3^2 F_{02} + \Lambda_0^0\Lambda_3^3 F_{03} \\
&+ \Lambda_0^1\Lambda_3^0 F_{10} + \Lambda_0^1\Lambda_3^1 F_{11} + \Lambda_0^1\Lambda_3^2 F_{12} + \Lambda_0^1\Lambda_3^3 F_{13} \\
&+ \Lambda_0^2\Lambda_3^0 F_{20} + \Lambda_0^2\Lambda_3^1 F_{21} + \Lambda_0^2\Lambda_3^2 F_{22} + \Lambda_0^2\Lambda_3^3 F_{23} \\
&+ \Lambda_0^3\Lambda_3^0 F_{30} + \Lambda_0^3\Lambda_3^1 F_{31} + \Lambda_0^3\Lambda_3^2 F_{32} + \Lambda_0^3\Lambda_3^3 F_{33} \\
&= \Lambda_0^0\Lambda_3^3 F_{03} + \Lambda_0^1\Lambda_3^3 F_{13} \\
F'_{03} &= \gamma\Big[F_{03} - \gamma\beta F_{13}\Big] \qquad (31)
\end{aligned}$$

fixando $\mu = 1$ *e fazendo variar* $\nu = 0, ..., 3$
para $\nu = 0$

$$F'_{10} = -F'_{01} = \gamma^2 \Big[\beta^2 - 1\Big] F_{01}$$

para $\nu = 1$

$$\begin{aligned} F'_{11} &= \Lambda^0_1\Lambda^0_1 F_{00} + \Lambda^0_1\Lambda^1_1 F_{01} + \Lambda^0_1\Lambda^2_1 F_{02} + \Lambda^0_1\Lambda^3_1 F_{03} \\ &+ \Lambda^1_1\Lambda^0_1 F_{10} + \Lambda^1_1\Lambda^1_1 F_{11} + \Lambda^1_1\Lambda^2_1 F_{12} + \Lambda^1_1\Lambda^3_1 F_{13} \\ &+ \Lambda^2_1\Lambda^0_1 F_{20} + \Lambda^2_1\Lambda^1_1 F_{21} + \Lambda^2_1\Lambda^2_1 F_{22} + \Lambda^2_1\Lambda^3_1 F_{23} \\ &+ \Lambda^3_1\Lambda^0_1 F_{30} + \Lambda^3_1\Lambda^1_1 F_{31} + \Lambda^3_1\Lambda^2_1 F_{32} + \Lambda^3_1\Lambda^3_1 F_{33} \\ &= \Lambda^0_1\Lambda^0_1 F_{00} + \Lambda^0_1\Lambda^1_1 F_{01} + \Lambda^1_1\Lambda^0_1 F_{10} + \Lambda^1_1\Lambda^1_1 F_{11} \\ F'_{11} &= 0 \end{aligned} \tag{32}$$

para $\nu = 2$

$$\begin{aligned} F'_{12} &= \Lambda^0_1\Lambda^0_2 F_{00} + \Lambda^0_1\Lambda^1_2 F_{01} + \Lambda^0_1\Lambda^2_2 F_{02} + \Lambda^0_1\Lambda^3_2 F_{03} \\ &+ \Lambda^1_1\Lambda^0_2 F_{10} + \Lambda^1_1\Lambda^1_2 F_{11} + \Lambda^1_1\Lambda^2_2 F_{12} + \Lambda^1_1\Lambda^3_2 F_{13} \\ &+ \Lambda^2_1\Lambda^0_2 F_{20} + \Lambda^2_1\Lambda^1_2 F_{21} + \Lambda^2_1\Lambda^2_2 F_{22} + \Lambda^2_1\Lambda^3_2 F_{23} \\ &+ \Lambda^3_1\Lambda^0_2 F_{30} + \Lambda^3_1\Lambda^1_2 F_{31} + \Lambda^3_1\Lambda^2_2 F_{32} + \Lambda^3_1\Lambda^3_2 F_{33} \\ &= \Lambda^1_1\Lambda^2_2 F_{12} \\ F'_{12} &= \gamma F_{12} \end{aligned} \tag{33}$$

para $\nu = 3$

$$\begin{aligned} F'_{13} &= \Lambda^0_1\Lambda^0_3 F_{00} + \Lambda^0_1\Lambda^1_3 F_{01} + \Lambda^0_1\Lambda^2_3 F_{02} + \Lambda^0_1\Lambda^3_3 F_{03} \\ &+ \Lambda^1_1\Lambda^0_3 F_{10} + \Lambda^1_1\Lambda^1_3 F_{11} + \Lambda^1_1\Lambda^2_3 F_{12} + \Lambda^1_1\Lambda^3_3 F_{13} \\ &+ \Lambda^2_1\Lambda^0_3 F_{20} + \Lambda^2_1\Lambda^1_3 F_{21} + \Lambda^2_1\Lambda^2_3 F_{22} + \Lambda^2_1\Lambda^3_3 F_{23} \\ &+ \Lambda^3_1\Lambda^0_3 F_{30} + \Lambda^3_1\Lambda^1_3 F_{31} + \Lambda^3_1\Lambda^2_3 F_{32} + \Lambda^3_1\Lambda^3_3 F_{33} \\ &= \Lambda^0_1\Lambda^3_3 F_{03} + \Lambda^1_1\Lambda^3_3 F_{13} \\ F'_{13} &= -\gamma\beta F_{03} + F_{13} \end{aligned}$$

fixando $\mu = 2$ *e fazendo variar* $\nu = 0, ..., 3$
para $\nu = 0$

$$F'_{20} = -F'_{02} = \gamma \Big[\beta F_{12} - F_{02}\Big]$$

para $\nu = 1$

$$F'_{21} = -F'_{12} = -\gamma F_{12}$$

para $\nu = 2$

$$\begin{aligned} F'_{22} &= \Lambda^0_2\Lambda^0_1 F_{00} + \Lambda^0_2\Lambda^1_1 F_{01} + \Lambda^0_2\Lambda^2_1 F_{02} + \Lambda^0_2\Lambda^3_1 F_{03} \\ &+ \Lambda^1_2\Lambda^0_1 F_{10} + \Lambda^1_2\Lambda^1_1 F_{11} + \Lambda^1_2\Lambda^2_1 F_{12} + \Lambda^1_2\Lambda^3_1 F_{13} \\ &+ \Lambda^2_2\Lambda^0_1 F_{20} + \Lambda^2_2\Lambda^1_1 F_{21} + \Lambda^2_2\Lambda^2_1 F_{22} + \Lambda^2_2\Lambda^3_1 F_{23} \\ &+ \Lambda^3_2\Lambda^0_1 F_{30} + \Lambda^3_2\Lambda^1_1 F_{31} + \Lambda^3_2\Lambda^2_1 F_{32} + \Lambda^3_2\Lambda^3_1 F_{33} \\ F'_{22} &= 0 \end{aligned} \tag{34}$$

para $\nu = 3$

$$\begin{aligned}
F'_{23} &= \Lambda_2^0\Lambda_3^0F_{00} + \Lambda_2^0\Lambda_3^1F_{01} + \Lambda_2^0\Lambda_3^2F_{02} + \Lambda_2^0\Lambda_3^3F_{03} \\
&+ \Lambda_2^1\Lambda_3^0F_{10} + \Lambda_2^1\Lambda_3^1F_{11} + \Lambda_2^1\Lambda_3^2F_{12} + \Lambda_2^1\Lambda_3^3F_{13} \\
&+ \Lambda_2^2\Lambda_3^0F_{20} + \Lambda_2^2\Lambda_3^1F_{21} + \Lambda_2^2\Lambda_3^2F_{22} + \Lambda_2^2\Lambda_3^3F_{23} \\
&+ \Lambda_2^3\Lambda_3^0F_{30} + \Lambda_2^3\Lambda_3^1F_{31} + \Lambda_2^3\Lambda_3^2F_{32} + \Lambda_2^3\Lambda_3^3F_{33} \\
&= \Lambda_2^2\Lambda_3^3F_{23} \\
F'_{23} &= F_{23} \qquad (35)
\end{aligned}$$

fixando $\mu = 3$ *e fazendo variar* $\nu = 0, \ldots, 3$
para $\nu = 0$

$$F'_{30} = -F'_{03} = \gamma\Big[\gamma\beta F_{13} - F_{03}\Big]$$

para $\nu = 1$

$$F'_{31} = -F'_{13} = \gamma\beta F_{03} - F_{13}$$

para $\nu = 2$

$$F'_{32} = -F'_{23} = -F_{23}$$

para $\nu = 3$

$$\begin{aligned}
F'_{23} &= \Lambda_3^0\Lambda_3^0F_{00} + \Lambda_3^0\Lambda_3^1F_{01} + \Lambda_3^0\Lambda_3^2F_{02} + \Lambda_3^0\Lambda_3^3F_{03} \\
&+ \Lambda_3^1\Lambda_3^0F_{10} + \Lambda_3^1\Lambda_3^1F_{11} + \Lambda_3^1\Lambda_3^2F_{12} + \Lambda_3^1\Lambda_3^3F_{13} \\
&+ \Lambda_3^2\Lambda_3^0F_{20} + \Lambda_3^2\Lambda_3^1F_{21} + \Lambda_3^2\Lambda_3^2F_{22} + \Lambda_3^2\Lambda_3^3F_{23} \\
&+ \Lambda_3^3\Lambda_3^0F_{30} + \Lambda_3^3\Lambda_3^1F_{31} + \Lambda_3^3\Lambda_3^2F_{32} + \Lambda_3^3\Lambda_3^3F_{33} \\
F'_{33} &= 0 \qquad (36)
\end{aligned}$$

2.4 - Os campos elétrico **E** e magnético **B**, num sistema inércial K formam um ângulo $< \pi/2$. Determinar os módulos dos campos $\mathbf{E}'$ e $\mathbf{B}'$ num outro referecial K' onde o ângulo formado por ele é $\pi/4$.

....:: Solução ::..

$$\begin{aligned}
\mathbf{B}\cdot\mathbf{E} &= \mathbf{B}'\cdot\mathbf{E}' = B'E'\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}B'E' \\
(\mathbf{B}\cdot\mathbf{E})^2 &= (\frac{1}{\sqrt{2}}B'E')^2 \\
\mathbf{B}'^2 &= \frac{2(\mathbf{B}\cdot\mathbf{E})^2}{\mathbf{E}'^2} \qquad (37)
\end{aligned}$$

sabendo que,

$$\mathbf{B}^2 - \mathbf{E}^2 = invariante \qquad (38)$$
$$\mathbf{E}\cdot\mathbf{B} = invariante \qquad (39)$$

a invariância da eq. (38) significa que se as intensidades do campo elétrico **E** *e do campo magnético* **B** *são iguais num sistema de referência, eles continuam iguais em outro. Se num referencial em repouso* $|\mathbf{B}| > |\mathbf{E}|$ *ou vice-versa, eles continuam obedecendo a mesma relação em um referencial em movimento. A eq. (39) significa que se os campos* **E** *e* **B** *são mutuamente* *perpendiculares em um referencial, isto é,* $\mathbf{E}\cdot\mathbf{B}=0$, *então eles também são perpendiculares em outro referencial.*

das eqs. (37) e (38)

$$\begin{aligned}
\mathbf{B}'^2-\mathbf{E}'^2 &= \mathbf{B}^2-\mathbf{E}^2 \qquad (40)\\
\frac{2(\mathbf{B}\cdot\mathbf{E})^2}{\mathbf{E}'^2}-\mathbf{E}'^2 &= \mathbf{B}^2-\mathbf{E}^2\\
2(\mathbf{B}\cdot\mathbf{E})^2-\mathbf{E}'^4 &= (\mathbf{B}^2-\mathbf{E}^2)\mathbf{E}'^2\\
\mathbf{E}'^4+(\mathbf{B}^2-\mathbf{E}^2)\mathbf{E}'^2-2(\mathbf{B}\cdot\mathbf{E})^2 &= 0\\
\textit{usando a fórmula de Bhaskara: } x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \textit{e fazendo } x\equiv\mathbf{E}'^2\\
\mathbf{E}'^2 &= \frac{-(\mathbf{B}^2-\mathbf{E}^2)\pm\sqrt{(\mathbf{B}^2-\mathbf{E}^2)^2-4(1)(-2(\mathbf{B}\cdot\mathbf{E})^2)}}{2(1)}\\
&= \frac{\mathbf{E}^2-\mathbf{B}^2\pm\sqrt{(\mathbf{B}^2-\mathbf{E}^2)^2+8(\mathbf{B}\cdot\mathbf{E})^2}}{2}\\
E' &= \frac{1}{\sqrt{2}}\left[\mathbf{E}^2-\mathbf{B}^2+\sqrt{(\mathbf{B}^2-\mathbf{E}^2)^2+8(\mathbf{B}\cdot\mathbf{E})^2}\right]^{1/2} \qquad (41)
\end{aligned}$$

para obtermos B', *basta proceder de modo análogo ao feito com* E' *na eq. (37),*

$$B' = \frac{1}{\sqrt{2}}\left[\mathbf{B}^2-\mathbf{E}^2+\sqrt{(\mathbf{B}^2-\mathbf{E}^2)^2+8(\mathbf{B}\cdot\mathbf{E})^2}\right]^{1/2} \qquad (42)$$

2.5 - Demonstrar as identidades abaixo

$$\begin{aligned}
\mathbf{C}\cdot grad(\mathbf{A}\cdot\mathbf{B}) &= \mathbf{A}\cdot(\mathbf{C}\cdot\nabla)\mathbf{B}+\mathbf{B}\cdot(\mathbf{C}\cdot\nabla)\mathbf{A} \qquad (43)\\
(\mathbf{C}\cdot\nabla)(\mathbf{A}\times\mathbf{B}) &= \mathbf{A}\times(\mathbf{C}\cdot\nabla)\mathbf{B}-\mathbf{B}\times(\mathbf{C}\cdot\nabla)\mathbf{A} \qquad (44)\\
(\nabla\times\mathbf{A})\times\mathbf{B} &= \mathbf{A}(\nabla\cdot\mathbf{B})-(\mathbf{A}\cdot\nabla)\mathbf{B}-\mathbf{A}\times(\nabla\times\mathbf{B})-\mathbf{B}\times(\nabla\times\mathbf{A}) \qquad (45)
\end{aligned}$$

....:: Solução (43) ::..

operação diferencial,

$$\mathbf{C}\cdot grad(\mathbf{A}\cdot\mathbf{B}) = \mathbf{C}\cdot\left[\nabla(\mathbf{A}\cdot\check{\mathbf{B}})\right]+\mathbf{C}\cdot\left[\nabla(\check{\mathbf{A}}\cdot\mathbf{B})\right] \qquad (46)$$

operação vetorial,

$$\begin{aligned}
\mathbf{A}\times(\check{\mathbf{B}}\times\nabla) &= -\nabla(\mathbf{A}\cdot\check{\mathbf{B}})+\check{\mathbf{B}}(\nabla\cdot\mathbf{A}) \\
\nabla(\mathbf{A}\cdot\check{\mathbf{B}}) &= \mathbf{A}\times(\nabla\times\mathbf{B})+(\mathbf{A}\cdot\nabla)\mathbf{B} \\
\mathbf{C}\cdot\big[\nabla(\mathbf{A}\cdot\check{\mathbf{B}})\big] &= \mathbf{C}\cdot\big[\mathbf{A}\times(\nabla\times\mathbf{B})+(\mathbf{A}\cdot\nabla)\mathbf{B})\big] \\
&= \mathbf{C}\cdot\big[\mathbf{A}\times(\nabla\times\mathbf{B})\big]+\mathbf{C}\cdot(\mathbf{A}\cdot\nabla)\mathbf{B} \\
&= (\mathbf{C}\times\mathbf{A})\cdot(\nabla\times\mathbf{B})+\mathbf{C}\cdot(\mathbf{A}\cdot\nabla)\mathbf{B} \\
&= (\mathbf{C}\cdot\nabla)(\mathbf{A}\cdot\mathbf{B})-(\mathbf{C}\cdot\mathbf{B})(\mathbf{A}\cdot\nabla)+\mathbf{C}\cdot(\mathbf{A}\cdot\nabla)\mathbf{B} \\
&= \mathbf{A}\cdot(\mathbf{C}\cdot\nabla)\mathbf{B}-\mathbf{C}\cdot(\mathbf{A}\cdot\nabla)\mathbf{B}+\mathbf{C}\cdot(\mathbf{A}\cdot\nabla)\mathbf{B} \\
&= \mathbf{A}\cdot(\mathbf{C}\cdot\nabla)\mathbf{B} \qquad (47)
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}\times(\nabla\times\check{\mathbf{A}}) &= -\check{\mathbf{A}}(\mathbf{B}\cdot\nabla)+\nabla(\check{\mathbf{A}}\cdot\mathbf{B}) \\
\nabla(\check{\mathbf{A}}\cdot\mathbf{B}) &= \mathbf{B}\times(\nabla\times\mathbf{A})+(\mathbf{B}\cdot\nabla)\mathbf{A} \\
\mathbf{C}\cdot\big[\nabla(\check{\mathbf{A}}\cdot\mathbf{B})\big] &= \mathbf{C}\cdot\big[\mathbf{B}\times(\nabla\times\mathbf{A})+(\mathbf{B}\cdot\nabla)\mathbf{A}\big] \\
&= \mathbf{C}\cdot\big[\mathbf{B}\times(\nabla\times\mathbf{A})\big]+\mathbf{C}\cdot(\mathbf{B}\cdot\nabla)\mathbf{A} \\
&= (\mathbf{C}\times\mathbf{B})\cdot(\nabla\times\mathbf{A})+\mathbf{C}\cdot(\mathbf{B}\cdot\nabla)\mathbf{A} \\
&= (\mathbf{C}\cdot\nabla)(\mathbf{B}\cdot\mathbf{A})-(\mathbf{C}\cdot\mathbf{A})(\mathbf{B}\cdot\nabla)+\mathbf{C}\cdot(\mathbf{B}\cdot\nabla)\mathbf{A} \\
&= \mathbf{B}\cdot(\mathbf{C}\cdot\nabla)\mathbf{A}-\mathbf{C}\cdot(\mathbf{B}\cdot\nabla)\mathbf{A}+\mathbf{C}\cdot(\mathbf{B}\cdot\nabla)\mathbf{A} \\
&= \mathbf{B}\cdot(\mathbf{C}\cdot\nabla)\mathbf{A} \qquad (48)
\end{aligned}$$

das eqs. (46), (47) e (48),

$$\mathbf{C}\cdot grad(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\cdot(\mathbf{C}\cdot\nabla)\mathbf{B}+\mathbf{B}\cdot(\mathbf{C}\cdot\nabla)\mathbf{A} \qquad (49)$$

....:: Solução (44) ::...

operação diferencial,

$$(\mathbf{C}\cdot\nabla)(\mathbf{A}\times\mathbf{B}) = (\mathbf{C}\cdot\nabla)(\mathbf{A}\times\check{\mathbf{B}})+(\mathbf{C}\cdot\nabla)(\check{\mathbf{A}}\times\mathbf{B}) \qquad (50)$$

operação vetorial,

$$\begin{aligned}
(\mathbf{C}\cdot\nabla)(\mathbf{A}\times\check{\mathbf{B}}) &= \mathbf{A}\times(\mathbf{C}\cdot\nabla)\mathbf{B} \qquad (51) \\
(\mathbf{C}\cdot\nabla)(\check{\mathbf{A}}\times\mathbf{B}) &= -\mathbf{B}\times(\mathbf{C}\cdot\nabla)\mathbf{A} \qquad (52)
\end{aligned}$$

das eqs. (50), (51) e (52),

$$(\mathbf{C}\cdot\nabla)(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\times(\mathbf{C}\cdot\nabla)\mathbf{B}-\mathbf{B}\times(\mathbf{C}\cdot\nabla)\mathbf{A} \qquad (53)$$

....:: Solução (45) ::...

operação diferencial,

$$(\nabla\times\mathbf{A})\times\mathbf{B} = (\nabla\times\mathbf{A})\times\check{\mathbf{B}}+(\nabla\times\check{\mathbf{A}})\times\mathbf{B} \qquad (54)$$

operação vetorial,

$$\begin{aligned}
(\nabla\times\mathbf{A})\times\check{\mathbf{B}} &= -\check{\mathbf{B}}\times(\nabla\times\mathbf{A}) = +\mathbf{A}(\check{\mathbf{B}}\cdot\nabla)-\nabla(\check{\mathbf{B}}\cdot\mathbf{A}) \\
&= \mathbf{A}(\nabla\cdot\mathbf{B})-\nabla(\check{\mathbf{B}}\cdot\mathbf{A}) \qquad (55)
\end{aligned}$$

$$\begin{aligned}
\mathbf{A}\times(\nabla\times\mathbf{B}) &= -\mathbf{B}(\mathbf{A}\cdot\nabla)+\nabla(\check{\mathbf{B}}\cdot\mathbf{A}) \\
\nabla(\check{\mathbf{B}}\cdot\mathbf{A}) &= \mathbf{A}\times(\nabla\times\mathbf{B})+(\mathbf{A}\cdot\nabla)\mathbf{B} \qquad (56)
\end{aligned}$$

$$
\begin{aligned}
(\nabla \times \check{\mathbf{A}}) \times \mathbf{B} &= -\mathbf{B} \times (\nabla \times \check{\mathbf{A}}) = +\check{\mathbf{A}}(\mathbf{B} \cdot \nabla) - \nabla(\mathbf{B} \cdot \check{\mathbf{A}}) \\
&= (\mathbf{B} \cdot \nabla)\mathbf{A} - \nabla(\check{\mathbf{A}} \cdot \mathbf{B})
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
\mathbf{B} \times (\nabla \times \mathbf{A}) &= -\mathbf{A}(\mathbf{B} \cdot \nabla) + \nabla(\check{\mathbf{A}} \cdot \mathbf{B}) \\
\nabla(\check{\mathbf{A}} \cdot \mathbf{B}) &= \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A}
\end{aligned} \tag{58}
$$

das eqs. (54), (55), (56), (57) e (58),

$$
(\nabla \times \mathbf{A}) \times \mathbf{B} = \mathbf{A}(\nabla \cdot \mathbf{B}) - (\mathbf{A} \cdot \nabla)\mathbf{B} - \mathbf{A} \times (\nabla \times \mathbf{B}) - \mathbf{B} \times (\nabla \times \mathbf{A}) \tag{59}
$$
